Forecasting the Yield Curve with Macroeconomic Variables

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EARF

Motivation

SGH

Motivation

Understanding movements in yields at dierent maturities crucial for:

- managing bond portfolios
- macroeconomic forecasting (Ang, Piazzesi, and Wei, 2006)
- better monetary policy (Brzoza-Brzezina and Kotlowski, 2014)

Motivation

Modeling and forecasting yields widely debated in the literature (see Piazzesi, 2010; Gurkaynak and Wright, 2012, for review)

- A lot of recognition for affine models
  [yields at dierent maturities as a linear function of few latent factors]

- Most popular model - latent factors follow an AR process
  (Diebold and Li, 2006)

- Model flexible enough to allow for the interactions between latent factors and macroeconomic variables
  (Ang and Piazzesi, 2003; Diebold, Rudebusch, and Aruoba, 2006).

Motivation

Our question:

Do macroeconomic time series help in forecasting the yield curve in the US.

Our contribution:

1. We confirm earlier results: forecasts from dynamic affine models tend to be more accurate than the benchmark (forward rates)

2. We show that even though latent factors are correlated with macroeconomic variables, models allowing for endogenous interactions between those factors and macroeconomic variables produce forecasts of worse quality

3. We show that yields forecasts conditional on the realization of macroeconomic variables are significantly more accurate than unconditional forecast.

Forecasting competition

Competing models: univariate benchmarks

Expectation hypothesis model - baseline:

$$R_{t,h}^{f}(m) = \frac{(m+h)R_t(m+h) - hR_t(h)}{m}$$

Random Walk - RW:

$$R_{t,h}^{f}(m) = R_t(m)$$

Autoregression AR(P):

$$R_{t,h}^{f}(m) = \alpha + \sum_{p=1}^{P} \rho_p R_{t,h-p}^{f}(m)$$

Latent factors / affine model of the yield curve

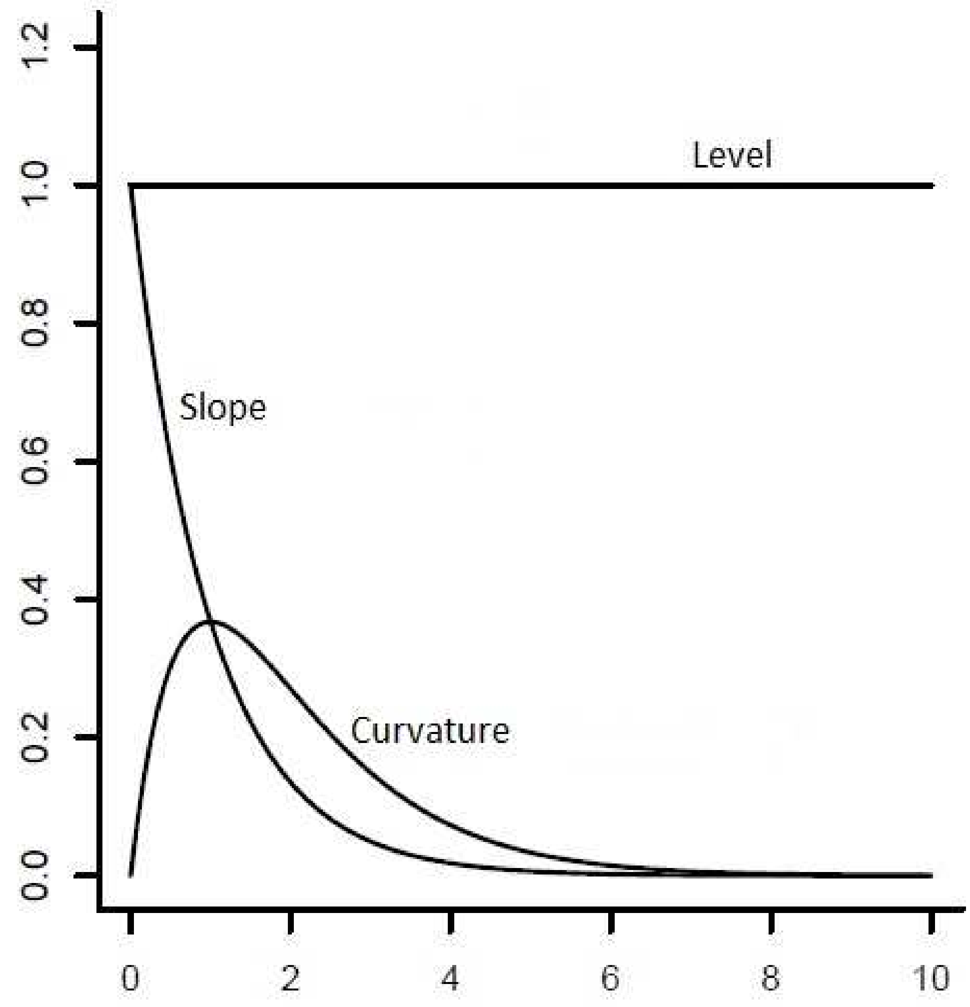
Nelson-Siegel (1987) model:

$$R(m) = L + S\left(\frac{1-e^{-m\lambda}}{m\lambda}\right) + C\left(\frac{1-e^{-m\lambda}}{m\lambda} - e^{-m\lambda}\right)$$

Diebold-Li (2006) model:

- Dynamic version of NS model
- *L*, *S* and *C* are time variant
- Parameter λ set to 0.0609 → loading on the curvature factor is maximum for m=2.5 years

$$R_t(m) = L_t + S_t\left(\frac{1-e^{-m\lambda}}{m\lambda}\right) + C_t\left(\frac{1-e^{-m\lambda}}{m\lambda} - e^{-m\lambda}\right)$$



Dynamic factor models

$$R_t(m) = L_t + S_t\left(\frac{1-e^{-m\lambda}}{m\lambda}\right) + C_t\left(\frac{1-e^{-m\lambda}}{m\lambda} - e^{-m\lambda}\right)$$

Diebold-Li AR(P):

Factors L_{t+h}^f, S_{t+h}^f and C_{t+h}^f are forecasted with AR(P) model

Diebold-Li VAR(P):

Factors L_{t+h}^f, S_{t+h}^f and C_{t+h}^f are forecasted with VAR(P) model

Diebold-Li BVAR(P):

Factors L_{t+h}^f, S_{t+h}^f and C_{t+h}^f are forecasted with BVAR(P) model

Forecast for the interest rate:

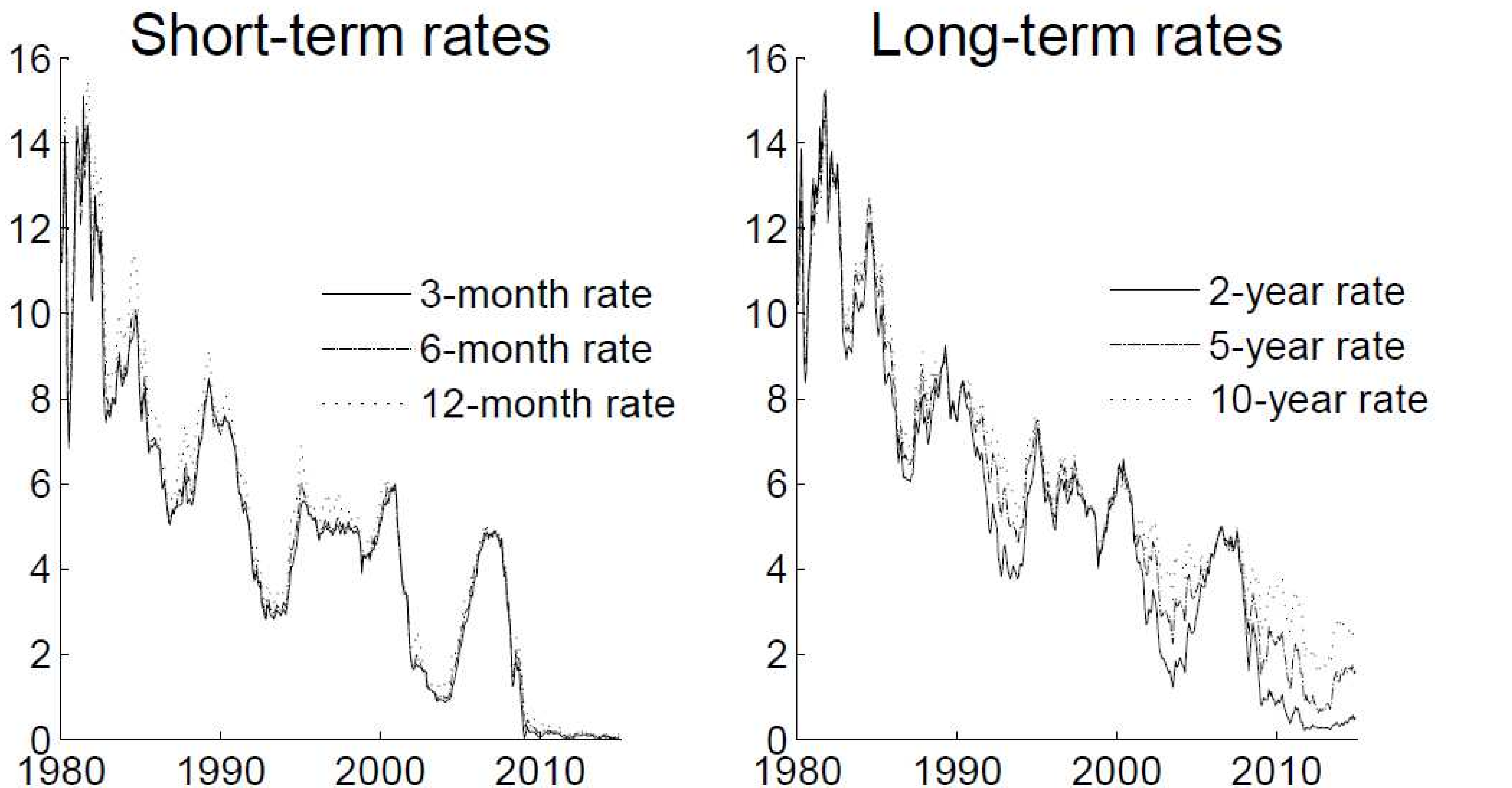
The values of interest rates at different maturities are then forecasted with formula

$$R_{t,h}^f(m) = L_{t,h}^f + S_{t,h}^f\left(\frac{1-e^{-m\lambda}}{m\lambda}\right) + C_{t,h}^f\left(\frac{1-e^{-m\lambda}}{m\lambda} - e^{-m\lambda}\right)$$

Data and forecast evaluation sample

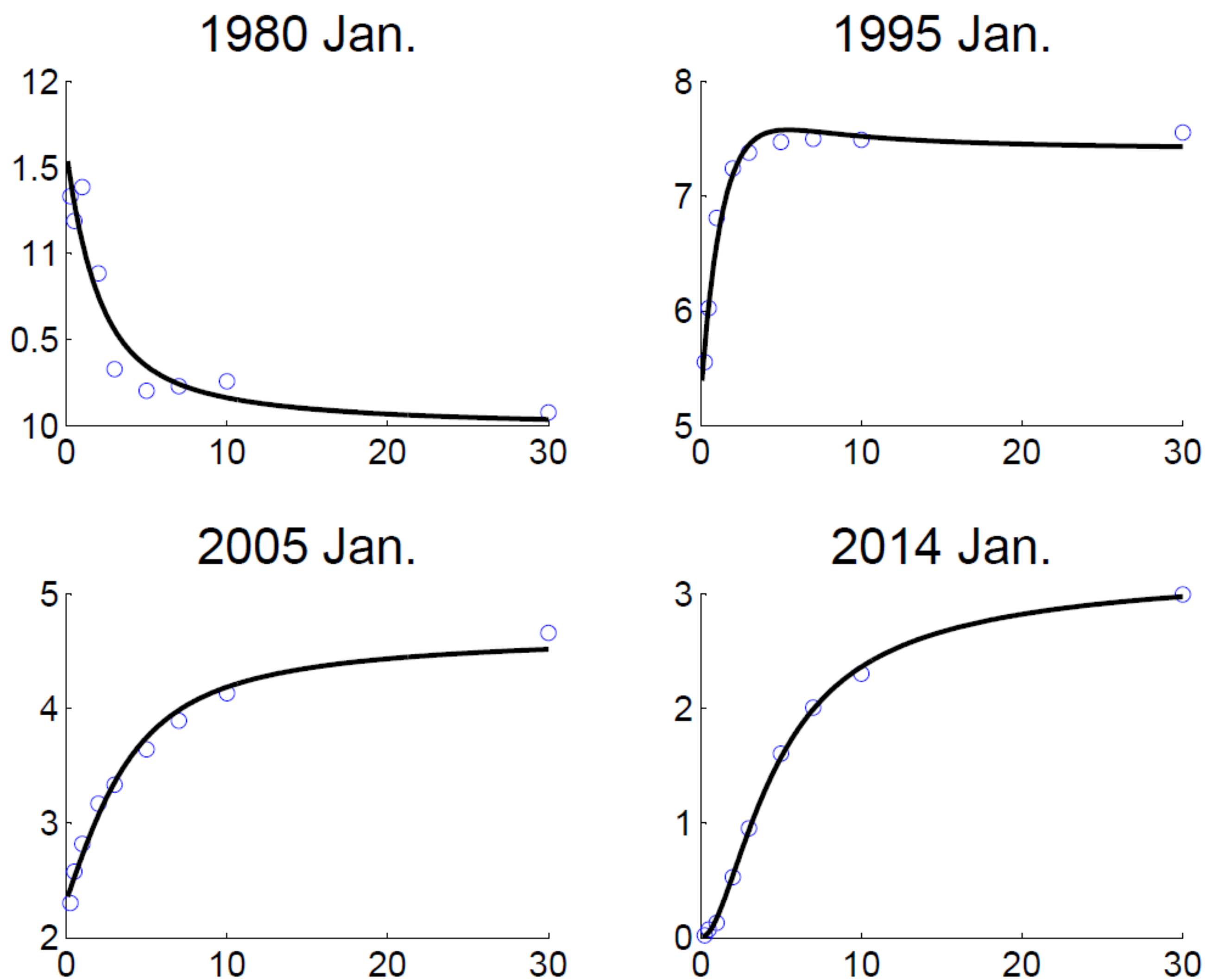
Observables:	Treasury bills (3M, 6M and 1Y) Treasury bonds (2Y, 3Y, 5Y, 7Y, 10Y and 30Y)
Sample:	1980:1 - 2014:11
Forecast evaluation:	2000:1-2014:11 179 obs. for 1m, 120 obs. for 60m ahead
Forecasting scheme:	rolling window of 240 months

Data



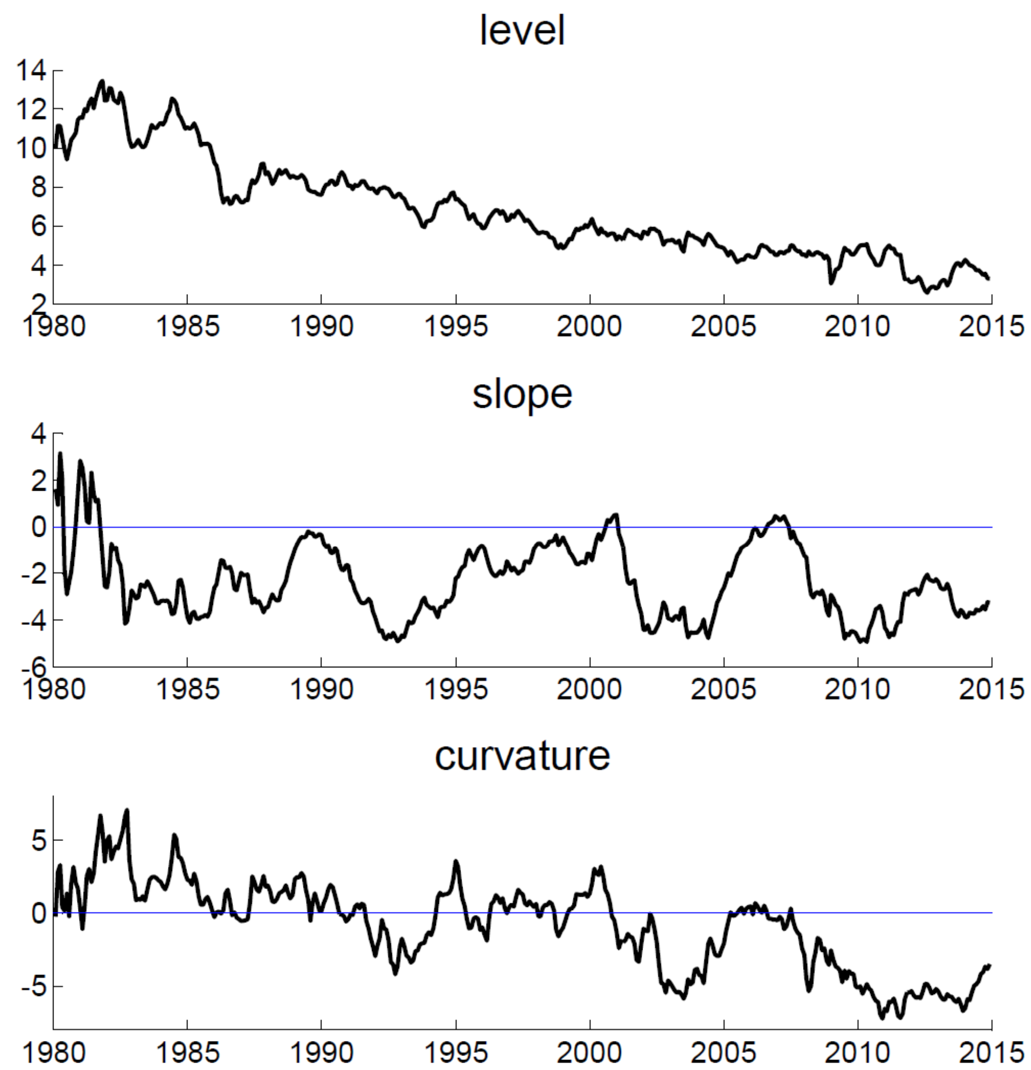
Source: Federal reserve.

Fit of Nelson-Siegel model



Latent factors

- Gradual decline of L_t
- The difference between LT and ST rates averaged to 2.35 pp (term premium, S_t)
- S_t correlated with business cycle fluctuations
- C_t relatively volatile and oscillates around zero.

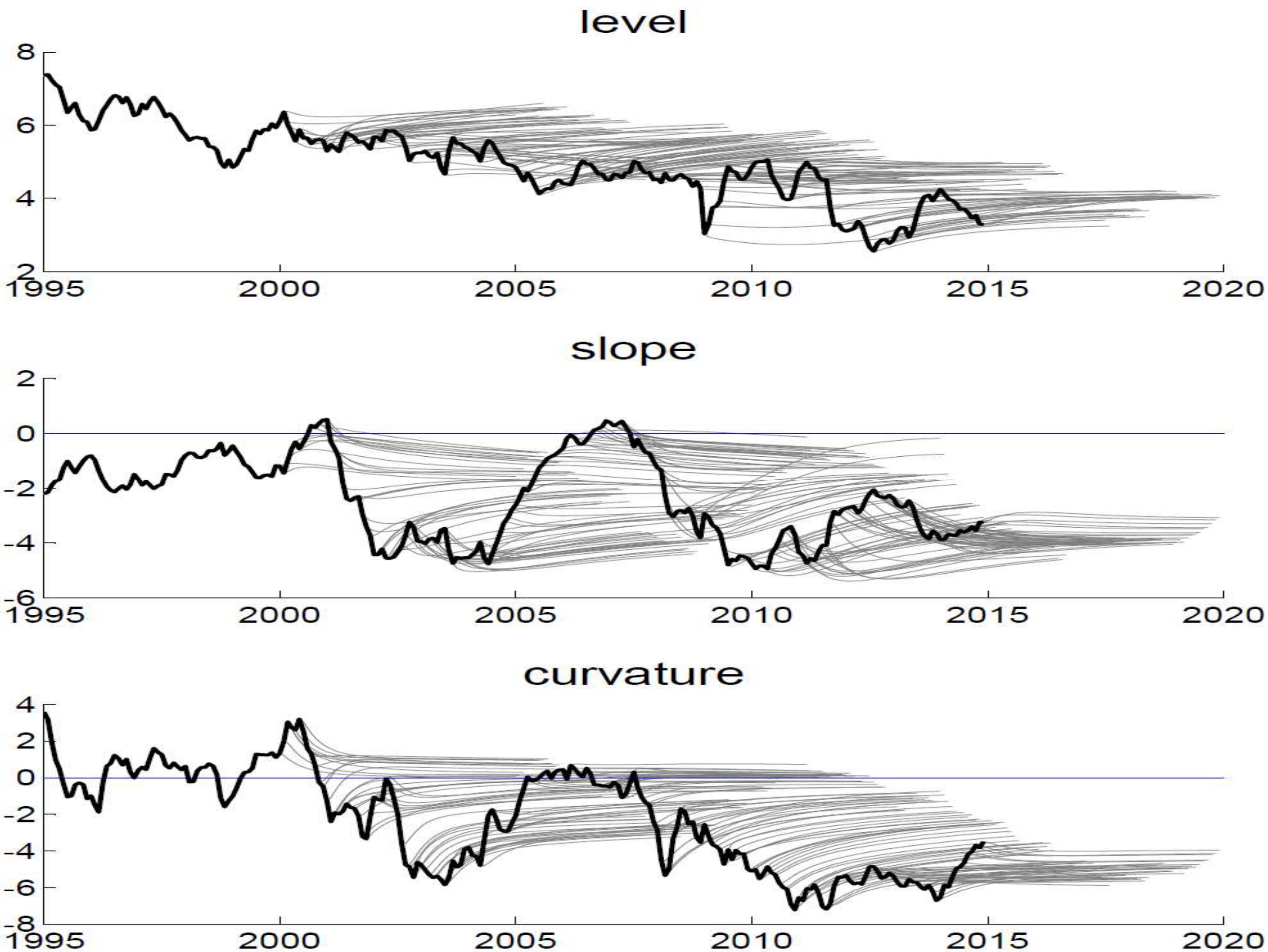


Mean Error

	1	3	6	12	24	60
	three-month yield					
Baseline	-0.09***	-0.17***	-0.35***	-0.86***	-1.89***	-3.36***
RW	-0.03	-0.09	-0.18	-0.40*	-0.71*	-1.28***
AR(1)	-0.02	-0.08	-0.16	-0.35	-0.62	-1.11**
AR(2)	-0.03**	-0.13**	-0.29**	-0.63***	-1.15***	-2.09***
AR(3)	-0.03**	-0.14***	-0.33***	-0.78***	-1.53***	-2.56***
DL-AR(1)	-0.12***	-0.21***	-0.36***	-0.66***	-1.09***	-1.73***
DL-AR(2)	-0.12***	-0.26***	-0.48***	-0.89***	-1.47***	-2.19***
DL-AR(6)	-0.12***	-0.24***	-0.45***	-0.83***	-1.37***	-2.01***
DL-VAR(1)	-0.07***	-0.08	-0.14	-0.35	-0.86**	-1.99***
DL-VAR(2)	-0.08***	-0.14**	-0.26**	-0.62***	-1.24***	-2.39***
DL-VAR(6)	-0.08***	-0.13***	-0.24**	-0.57***	-1.30***	-2.37***
DL-BVAR(1)	-0.07***	-0.08	-0.13	-0.34	-0.83**	-1.97***
DL-BVAR(2)	-0.07***	-0.09	-0.17	-0.42*	-0.95**	-2.06***
DL-BVAR(6)	-0.08***	-0.10*	-0.19*	-0.48**	-1.06***	-2.14***

- for all models and maturities forecasts are significantly biased
- the bias largest for baseline model (term premium effect?)
- one-lag models seems to perform somewhat better

Forecasts from AR(1): problems with the level factor



Root Mean Squarred Error

	1	3	6	12	24	60
			three-month yield			
Baseline	0.20	0.46	0.82	1.51	2.62	3.67
RW	0.94	0.99	0.96	0.94	0.90*	0.72***
AR(1)	0.96	1.01	0.99	0.96	0.94	0.69***
AR(2)	0.83*	0.94	0.96	0.95	0.92**	0.73***
AR(3)	0.83**	0.94	0.91*	0.91**	0.91***	0.82***
DL-AR(1)	1.16***	1.06*	0.98	0.89**	0.81***	0.70***
DL-AR(2)	1.10*	1.08**	1.04	0.94	0.82***	0.73***
DL-AR(6)	1.09	1.07	1.00	0.88*	0.76***	0.67***
DL-VAR(1)	0.98	0.98	0.96	0.92**	0.90**	0.77***
DL-VAR(2)	0.92*	0.93**	0.96*	0.93**	0.91**	0.82***
DL-VAR(6)	0.97	0.94	0.88**	0.86***	0.88***	0.76***
DL-BVAR(1)	0.98	0.98	0.96	0.92**	0.90**	0.76***
DL-BVAR(2)	0.95*	0.97	0.96	0.92*	0.92**	0.79***
DL-BVAR(6)	0.94*	0.91**	0.90***	0.89***	0.89***	0.76***

- forecasts from the baseline tend to be worst (term premium effect?)
- In comparison to RW: no sizeable improvement for AR and dynamic affine models

Adding macroeconomic variables

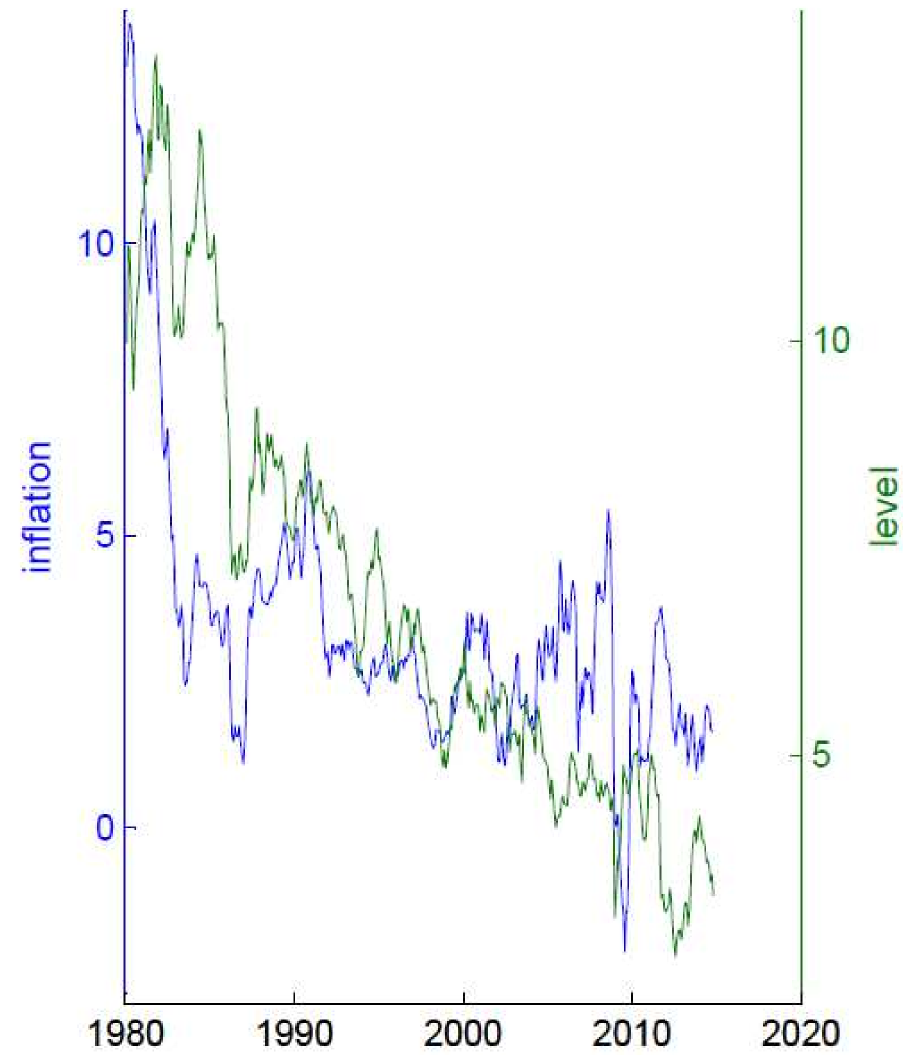
Latent factors and macroeconomic variables

Diebold, Rudebusch, and Aruoba (2006):

Level factor ←→ inflation

Logic:

Level factor =
LT real rate + LT expected inflation



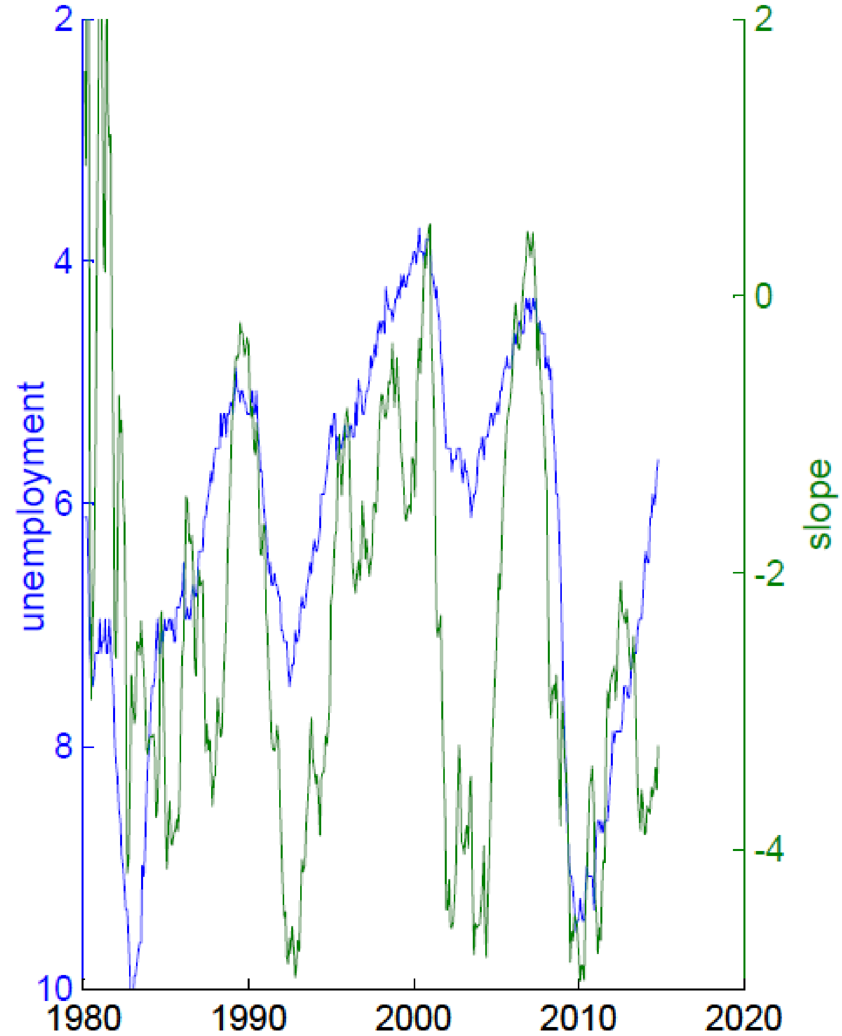
Latent factors and macroeconomic variables

Diebold, Rudebusch, and Aruoba (2006):

slope factor ←→ output fluctuations

Logic:

Slope fact. driven by monetary policy, hence captures the cyclical response of the central bank



Dynamic factor models with macro- vars. ENODOGENOUS MODELS

Diebold-Li ARXendo(P):

L_{t+h}^{f} is calculated with VAR(P) model for $[L_t \ \pi_t]$;
S_{t+h}^{f} is calculated with VAR(P) for $[S_t \ u_t]$;
C_{t+h}^{f} is calculated with AR(P)

Diebold-Li VARXendo(P):

Factors L_{t+h}^{f}, S_{t+h}^{f} and C_{t+h}^{f} are forecasted with VAR(P) model for $[L_t \ S_t \ c_t \ \pi_t \ u_t]$

Diebold-Li BVARXendo(P):

Factors L_{t+h}^{f}, S_{t+h}^{f} and C_{t+h}^{f} are forecasted with BVAR(P) model for $[L_t \ S_t \ c_t \ \pi_t \ u_t]$

Forecast for the interest rate:

The values of interest rates at different maturities are then forecasted with formula

$$R_{t,h}^{f}(m) = L_{t,h}^{f} + S_{t,h}^{f}\left(\frac{1-e^{-m\lambda}}{m\lambda}\right) + C_{t,h}^{f}\left(\frac{1-e^{-m\lambda}}{m\lambda} - e^{-m\lambda}\right)$$

Dynamic factor models with macro- vars.
EXOGENOUS MODELS

Diebold-Li ARXegzo(P):

L^f_{t+h} is calculated with ARX(P) conditional on realization for π_{t+h}
S^f_{t+h} is calculated with ARX(P) conditional on realization for u_{t+h}
C^f_{t+h} is calculated with AR(P)

Diebold-Li VARXegzo(P) / Diebold-Li BVARXegzo(P)

Factors L^f_{t+h}, S^f_{t+h} and C^f_{t+h} are forecasted with VARX(P)/BVAR(P) model $[L_t\ S_t\ c_t]$ conditional on realization for $[\pi_{t+h}\ u_{t+h}]$

Forecast for the interest rate:

The values of interest rates at different maturities are then forecasted with formula

$$R^f_{t,h}(m) = L^f_{t,h} + S^f_{t,h}\left(\frac{1-e^{-m\lambda}}{m\lambda}\right) + C^f_{t,h}\left(\frac{1-e^{-m\lambda}}{m\lambda} - e^{-m\lambda}\right)$$

Root Mean Squarred Error

	1	3	6	12	24	60
	three-month yield					
DL-AR(1)	0.23	0.49	0.81	1.35	2.13	2.59
DL-ARXendo(1)	1.01	1.07	1.12*	1.17**	1.21***	1.51*
DL-VARXendo(1)	0.86***	0.96	1.03	1.10	1.20***	1.15**
DL-BVARXendo(1)	0.86***	0.95	1.02	1.10	1.20***	1.14**
DL-ARXegzo(1)	1.01	1.06	1.08	1.05	0.89	0.67**
DL-VARXegzo(1)	0.85***	0.94	1.00	1.00	0.89	0.61***
DL-BVARXegzo(1)	0.85***	0.94	1.00	1.01	0.90	0.61***
	two-year yield					
DL-AR(1)	0.22	0.48	0.75	1.20	1.91	2.59
DL-ARXendo(1)	1.03	1.05*	1.08*	1.10*	1.10	1.18***
DL-VARXendo(1)	1.10***	1.11**	1.11**	1.10	1.14**	1.13***
DL-BVARXendo(1)	1.09***	1.10**	1.11**	1.10	1.14**	1.13***
DL-ARXegzo(1)	1.02	1.04	1.05	1.02	0.87**	0.64***
DL-VARXegzo(1)	1.10***	1.10**	1.08	0.99	0.86**	0.75***
DL-BVARXegzo(1)	1.09***	1.09**	1.08	1.00	0.87**	0.74***
	ten-year yield					
DL-AR(1)	0.23	0.43	0.61	0.80	1.11	1.76
DL-ARXendo(1)	1.02*	1.03*	1.05*	1.06	1.07	1.10**
DL-VARXendo(1)	1.02	1.03	1.05	1.08	1.24**	1.31**
DL-BVARXendo(1)	1.01	1.03	1.04	1.07	1.24**	1.31**
DL-ARXegzo(1)	1.02**	1.05*	1.08**	1.10**	1.02	0.90
DL-VARXegzo(1)	1.03*	1.07**	1.11*	1.14*	1.15*	1.24***
DL-BVARXegzo(1)	1.03	1.06**	1.11*	1.13	1.15*	1.23***

Dynamic factor models with macro- vars.

- Allowing for endogenous interactions between macro vars. and latent factors deteriorates the accuracy of forecasts for yields at all maturities

- Conditional forecasts for ST and MT yields are generally more accurate than unconditional forecasts

- For LT rates the ``extra" information is of little help and may even deteriorate the quality of forecasts compared to DL-AR(1)

- Overall, our results show that allowing for the interaction of latent factors with macro. variables is of little help in forecasting the yields, unless we allow for an information advantage in the forecasting contest

Conclusions

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Conclusions

1. **Numerous advantages of dynamic affine models of the yield curve:** tractability, simplicity, intuition and good forecasting properties

2. **The dynamics of the latent factors: level, slope and curvature** AR(1) specification seems to be is a good choice

3. **Allowing for interactions between the factors (VAR) or richer lag structure (P>1) is counterproductive.** Gains from reacher specication are more than counterbalanced by the higher estimation errors

4. **Latent factors are strongly correlated with macroeconomic variables**, but it cannot be exploited in forecasting the yield curve unless we use forecasts conditional on the realization of macroeconomic variables